

# How to use multiple graphical representations to support conceptual learning? Research-based principles in the Fractions Tutor

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**Abstract.** Multiple graphical representations are ubiquitous in educational materials because they serve complementary roles in emphasizing conceptual aspects of the domain. Yet, to benefit robust learning, students have to understand each representation and make connections between them. We describe research-based principles for the use of multiple graphical representations within intelligent tutoring systems (ITSs). These principles are the outcome of a series of iterative classroom experiments with the Fractions Tutor with over 3,000 students. The implementation of these principles into the Fractions Tutor results in robust conceptual learning. To our knowledge, the Fractions Tutor is the first ITS to use multiple graphical representations by implementing research-based principles to support conceptual learning. The instructional design principles we established apply to ITSs across domains.

**Keywords:** Multiple graphical representations, ITSs, classroom evaluation

## 1 Introduction

Multiple graphical representations are used in all science and math domains [1] because they serve complementary roles to illustrate conceptual aspects of the domain content. Yet, multiple representations do not automatically enhance learning [2]). To benefit from them, students need to understand each individual representation, become fluent in using them, and make connections between them.

ITSs provide novel opportunities for supporting students' learning with multiple graphical representations because they can provide individualized support for students' interactions with the representations. However, these opportunities are under-researched, leaving developers of ITSs without guidance on how best to implement instructional support for learning with multiple graphical representations.



**Fig. 1.** Interactive representations used in Fractions Tutor: circle, rectangle, number line.

## **2 Principles for the Use of Multiple Graphical Representations**

In this paper, we present a set of instructional design principles for the effective use of multiple graphical representations within ITSs. These principles are the outcome of a sequence of classroom experiments with over 3,000 students in grades 4-6. As part of these experiments, we iteratively improved an ITS for fractions that focuses on conceptual learning [3]. The Fractions Tutor uses multiple interactive, abstract graphical representations (see Fig. 1), provided in addition to text and symbols.

### **2.1 Use Multiple Graphical Representations to Support Conceptual Learning**

A vast literature documents the advantages of dual representations on students' learning [2]: text paired with one graphical representation leads to better learning than text alone. However, it remains an open question whether this advantage extends to *multiple graphical* representations compared to a *single graphical* representation, each provided in addition to text and symbols.

In several experiments, we found that multiple graphical representations lead to better learning of robust conceptual knowledge [3-5], compared to a single graphical representation. Yet, we also found that the advantage of multiple graphical representations on students' conceptual learning depends on what types of instructional support they receive to understand the individual graphical representations, and to make connections between the graphical representations.

### **2.2 Use Prompts to Support *Understanding* of Graphical Representations**

To benefit from multiple graphical representations, students need to conceptually understand how each graphical representation depicts information. We investigated the use of menu-based reflection prompts to support students in making sense of how each graphical representation depicts the concepts of numerator and denominator. In a classroom experiment with 132 students [6], we compared versions of the Fractions Tutor with or without such prompts. Results show that students only benefited from multiple graphical representations when reflection prompts were provided.

### **2.3 Interleave Topics to Support *Understanding* of Graphical Representations**

A vast literature documents the advantages of interleaving learning tasks [e.g., 7]: students learn better when frequently alternating between topics (e.g., when topics a and b are interleaved, a-b-a-b, rather than blocked, a-a-b-b). However, in multi-representational ITSs, problems can vary on two dimensions: topics and graphical representations. Should we interleave topics while blocking representations (e.g., a1-b1-a2-b2, where a and b are topics, and 1 and 2 are representations)? Or should we interleave representations while blocking topics (e.g., a1-a2-b1-b2)?

We investigated this question in a classroom experiment with 158 students [14]. Results show a significant advantage of interleaving topics while blocking graphical representations on students' understanding of graphical representations. This finding demonstrates that interleaving topics while blocking graphical representations is a further means to support students' understanding of graphical representations.

#### **2.4 Interleave Representations to Support *Fluency* with Graphical Representations**

Building on the previous experiment, we investigated whether combining interleaved practice with topics *and* interleaved practice with graphical representations supports students in developing fluency with individual graphical representations. Interleaving graphical representations requires students to repeatedly load their knowledge about each graphical representation from long-term memory into working memory. This should strengthen their knowledge about each graphical representation, help them recall this knowledge later on, and thereby promote fluency-building processes.

We investigated this hypothesis in a classroom experiment with 587 students [5]. All students worked on the same tutor problems which were provided in different sequences: graphical representations were either blocked or interleaved. Results show that students learn better when graphical representations are interleaved (in addition to topics being interleaved).

#### **2.5 Support *Connection-Making* between Multiple Graphical Representations**

Successful learning of conceptual knowledge of the domain depends on students' ability to make connections between multiple graphical representations. In a classroom experiment with 599 students, we investigated the complementary effects of two types of support for connection making on students' conceptual learning [3]. Sense-making support aims at helping students understand the correspondences between pairs of graphical representations (e.g., circle and number line) based on their structural components [8]. We implemented two types of sense-making support: worked examples [9] which required students to make connections between graphical representations themselves, and with auto-linked graphical representations, where the system automatically presented students with these correspondences [10]. Fluency-building support helps students gain experience in relating graphical representations based on their perceptual properties [11].

Our results demonstrate that only students who received both sense-making support and fluency-building support for connection-making benefited from multiple graphical representations. Furthermore, worked examples were the more effective type of support for sense-making of connections. Only the condition that received worked examples combined with fluency-building support significantly outperformed a single-representation control condition on conceptual knowledge of fractions.

### 3 Conclusions

We describe a set of instructional design principles for the effective use of multiple graphical representations within ITSs. These principles are the outcome of a series of controlled experiments conducted in real classrooms. The implementation of these principles in the Fractions Tutor results in robust learning of conceptual domain knowledge. Our research shows how the use of an ITS as a research platform can be instrumental to establishing instructional design principles.

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