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Making connections among multiple graphical representations of fractions: sense-making competencies enhance perceptual fluency, but not vice versa

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Abstract Prior research shows that representational competencies that enable students to use graphical representations to reason and solve tasks is key to learning in many science, technology, engineering, and mathematics domains. We focus on two types of representational competencies: (1) *sense making* of connections by verbally explaining how different representations map to one another, and (2) *perceptual fluency* that allows students to fast and effortlessly use perceptual features to make connections among representations. Because these different competencies are acquired via different types of learning processes, they require different types of instructional support: sense-making activities and fluency-building activities. In a prior experiment, we showed benefits for combining sense-making activities and fluency-building activities. In the current work, we test how to combine these two forms of instructional support, specifically, whether students should first work on sense-making activities or on fluency-building activities. This comparison allows us to investigate whether sense-making competencies enhance students' acquisition of perceptual fluency (sense-making-first hypothesis) or whether perceptual fluency enhances students' acquisition of sense-making competencies (fluency-first hypothesis). We conducted a lab experiment with 74 students from grades 3–5 working with an intelligent tutoring system for fractions. We assessed learning processes and learning outcomes related to representational competencies and domain knowledge. Overall, our results support the sense-making-first hypothesis, but not the fluency-first hypothesis.

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Instructional materials in science, technology, engineering, and mathematics (STEM) domains use graphical representations (GRs) to illustrate complex concepts. Typically, instruction uses *multiple* GRs because each emphasizes a different conceptual aspect of the domain knowledge. For example, circle diagrams show fractions as parts of an inherent whole, rectangles show fractions as parts of a whole that can be partitioned in multiple ways, and numberlines show fractions as measures of length (Fig. 1) (Kieren 1993; Post et al. 1982). Students can construct a deeper understanding of the domain knowledge by connecting the concepts shown by multiple GRs to one another (Schnotz 2005, 2014; Seufert 2003). Research shows that multiple GRs can enhance students' learning of domain knowledge (e.g., Ainsworth 2006; Ainsworth et al. 2002; Bodemer and Faust 2006).

However, multiple GRs can also hinder rather than help students (Rau et al. 2015). This phenomenon results from the so-called *representation dilemma* (Dreher and Kuntze 2014): On the one hand, students have to learn *about* GRs; for example, how circle diagrams show the numerator and denominator of a fraction. On the other hand, students learn domain knowledge *from* the GRs; for example, to learn what a fraction is, students may have to interpret circle diagrams that depict concrete scenarios (e.g., 1/4 of a cookie). This leads to a major educational challenge: how can students learn new domain knowledge from GRs without knowing how they show information, and—at the same time—learn to how GRs show concepts they have not yet learned?

Learning *about* representations means that students acquire *representational competencies:* knowledge and skills that enable them to use GRs to reason and solve tasks (Gilbert 2005, 2008; National Research Council, NRC 2006). Prior research has documented that representational competencies enable students to learn domain knowledge *from* GRs (Ainsworth 2008; de Jong et al. 1998; McElhaney et al. 2015; NRC 2006). As a result, research on learning with representations has investigated how to design instructional activities that support students' acquisition of representational competencies *while* they learn domain knowledge (Eilam and Ben-Peretz 2012). That is, while such instructional activities help students learn *about* the GRs (i.e., acquisition of representational competencies), the main goal is to help students learn *from* the GRs (i.e., acquisition of domain knowledge).

One important type of representational competencies is the ability to *make sense of connections* among multiple GRs by explaining differences and similarities between GRs (Rau 2016). For example, in Fig. 1, a student might make sense of connections between circle and numberline by relating the number of colored sections in the circle to the number of sections between 0 and the dot in the numberline, and the number of total sections in the circle to the number of sections between 0 and 1 in the numberline. Research on expertise suggests that a second type of representational competencies is important: perceptual fluency—the ability to use perceptual features to make connections among GRs automatically and with little mental effort (Rau 2016; Gibson

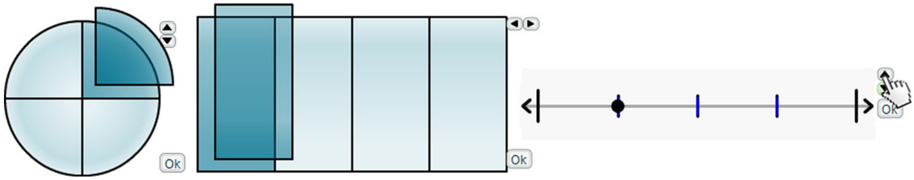


Fig. 1 Interactive graphical representations used in the Fractions Tutor: *circle*, *rectangle*, and *number line*. (Color figure online)

2000; Kellman and Massey 2013). For example, a student should see "at a glance" that the circle and numberline in Fig. 1 show the same fraction because they cover the same proportion of area or of length. As an analogy, perceptual fluency can be thought of as the ability to make connections by using "fast thinking" processes in Kahneman's description of "System 1" thinking in decision making. Sense-making competencies can be thought of as the student's ability to make connections using "deliberate thinking" processes or "System 2" thinking (Kahneman 2003). While these representational competencies are considered to be domain general learning strategies, students have to learn to engage in these strategies with domain-specific GRs (Eilam and Ben-Peretz 2012; NRC 2006).

Prior research suggests that sense-making competencies and perceptual fluency require different types of instructional support because they are acquired via different types of learning processes (Rau 2016; Koedinger et al. 2012). To help students acquire sense-making competencies, instructional activities—"*sense-making activities*"—should support students' engagement in explicit explanation-based processes (e.g., helping students explain that the circle diagram and the rectangle in Fig. 1 both show the same fraction because they show the same part colored out of the whole shape). To help students acquire fluency, instructional activities—"*fluency-building activities*"—should engage students in non-verbal, inductive pattern recognition processes (e.g., helping students see that the circle diagram and the rectangle in Fig. 1 both have the same proportion of area colored). Our own prior research shows that providing students with sense-making activities and fluency-building activities enhances their learning of domain knowledge (Rau et al. 2016). This finding raises the question of how sense-making activities and fluency-building activities should be combined. In this paper, we investigate in which sequence sense-making activities and fluency-building activities should be provided to students, so that support for representational competencies enhances students' learning of domain knowledge. To this end, we conducted an experiment that tested the effect of different sequences of sense-making activities and fluency-building activities on students' acquisition of domain knowledge. In addition, we used causal path analysis to test whether sense-making competencies enhance students' acquisition of fluency while working on fluency-building activities, and whether fluency enhances their acquisition of sense-making competencies.

Theoretical background

In this section, we review research suggesting that students' acquisition of sense-making competencies and fluency should be supported by different types of instructional activities because they are acquired via different types of learning processes. Then we review research about the interplay among these learning processes, which yields predictions about whether sense-making activities should be provided before fluency-building activities or vice versa.

Two types of representational competencies

Making sense of connections: competencies, processes, and instructional activities

Sense-making competencies describe the ability to explain mappings between different GRs using "deliberate thinking" processes (Kahneman 2003). Much prior research on learning with representations has focused sense-making competencies, albeit under a variety of terms. For example, Bodemer and Faust (2006) and Schnotz (2005, 2014) speak of integrating information from multiple representations by explaining mappings between different representations. Ainsworth (2006) discusses the importance of understanding how different representations may complement and constrain one another, which involves explaining mappings of visual features that show corresponding or different information. Seufert (2003) uses the term "coherence formation" to describe students' ability to connect different representations between corresponding features of the representations. We propose the term *sense-making competencies* to align with the cognitive psychology literature, which suggests that competencies that involve explaining of mappings across multiple entities are acquired via *sense-making processes* (Koedinger et al. 2012).

Sense-making processes are verbally mediated explanation-based processes by which students reason about principles (Koedinger et al. 2012). Sense-making competencies involve learning of general principles that describe which perceptual features in the GRs depict which concepts and how features of different GRs map onto one another (Ainsworth 2006; DeLoache 2000; Eilam and Ben-Peretz 2012; Schnotz 2014, 2005; Uttal and O'Doherty 2008). Learning these principles involves learning to distinguish perceptual features that show conceptually meaningful information from "surface features" that are conceptually irrelevant (Ainsworth 2006; Lowe 1993, 1994; Seufert 2003). Further, it involves learning how mappings of perceptual features to concepts apply to multiple situations (Gentner 1983; Gentner et al. 2003). For example, the color of circle and rectangle representations (see Fig. 1) is a surface feature because color cannot be used to determine the fraction. By contrast, the total number of sections is a relevant feature because it corresponds to the denominator of the fraction. Students may incorrectly assume that the blue color of a circle representation is conceptually relevant, and they may therefore incorrectly infer that any rectangle that is blue shows the same fraction, even if it is partitioned into fewer sections. Prior research shows that students learn to distinguish surface features from relevant features by explaining their mappings to concepts (Ainsworth et al. 2002; Rau et al. 2014a; Lowe 1993, 1994; Seufert 2003). Hence, the ability to explain how different representations map to one another based on corresponding and differing features and the ability to integrate information from multiple representations involves sense-making processes.

Sense-making competencies can enhance students' learning of domain knowledge (Ainsworth 2006; NRC 2006; Eilam and Ben-Peretz 2012). Specifically, when students engage in sense-making processes, they abstract away from the GRs and extract conceptual understanding of the underlying principles (e.g., a fraction is a quantity relative to a unit). Therefore, sense-making processes are an important mechanism through which students learn domain knowledge from multiple GRs. Sense-making competencies are recognized as an important aspect of domain expertise (Dreyfus and Dreyfus 1986; Richman et al. 1996), and as an important educational goal in the literature on learning with representations (Ainsworth 2006; Patel and Dexter 2014), in science education (Jones et al. 2005; Wu and Shah 2004), and in math education (Charalambous and Pitta-Pantazi 2007; Cramer 2001; Kaput 1987). Furthermore, educational practice guides emphasize the importance of helping students acquire sense-making competencies (National Council of Teachers of Mathematics, NCTM 2000, 2006).

In designing *sense-making activities* for our research, our goal was to create instructional activities that—besides enhancing sense-making competencies—enhance students' learning of domain knowledge. Because sense-making processes are an important mechanism through which students learn domain knowledge, helping students engage in sense-making processes should enhance their learning of domain knowledge while they work on sense-making activities. Further, sense-making competencies should accelerate students' learning of domain knowledge from GRs in subsequent instructional activities.

To achieve this goal, we draw on prior research that has yielded a number of principles for the instructional design of sense-making activities (Ainsworth 2006; Ainsworth and Loizou 2003; Berthold et al. 2008; Berthold and Renkl 2009; Bodemer and Faust 2006; van der Meij and de Jong 2006, 2011). First, sense-making activities should ask students to *verbally explain* which perceptual features of the GRs depict corresponding concepts (i.e., similarities between the GRs) and which features show complementary information (i.e., differences between the GRs). For example, prompting students to self-explain mappings between GRs has been shown to enhance learning of domain knowledge in math (Berthold et al. 2008; Berthold and Renkl 2009), physics (Van der Meij and de Jong 2011), and biology (Seufert 2003; Seufert and Brünken 2006), computer sciences (Baetge and Seufert 2010).

Second, sense-making activities should ask students to *actively compare* GRs. Several experiments on math and science learning show that students who are asked to actively map perceptual features that show corresponding information across GRs show higher learning outcomes on domain knowledge tests than students who are presented with pre-made mappings of the features (Bodemer and Faust 2006; Bodemer et al. 2004, 2005; Gutwill et al. 1999; Özgün-Koca 2008; Stern et al. 2003).

Finally, prior research shows that students have a tendency to focus on surface features that are incidental rather than on conceptually relevant features (Ainsworth et al. 2002; Rau et al. 2014a; Lowe 1993, 1994; Seufert 2003). As mentioned, the color of circle and rectangle representations (see Fig. 1) is a surface feature, whereas the total number of sections is a relevant feature. Students may incorrectly assume that the blue color of a circle representation is conceptually relevant, and they may therefore incorrectly infer that any rectangle that is blue shows the same fraction, even if it is partitioned into fewer sections. Therefore, students need *assistance* in identifying relevant perceptual features. Such assistance can be provided in the form of feedback on students' explanations (Rau et al. 2015, 2016; Bodemer and Faust 2006; van der Meij and de Jong 2006) or color coding (Berthold and Renkl 2009; Ozcelik et al. 2009). Assistance is particularly important for students with low prior knowledge (Bodemer and Faust 2006; Stern et al. 2003) and when problems are particularly complex (van der Meij and de Jong 2006).

In sum, prior research suggests that sense-making activities that engage students in sense-making processes enhance their learning of domain knowledge.

Fluently making connections: competencies, processes, and instructional activities

Fluency in making connections among GRs describes the ability to automatically relate representations to one another, using processes akin to Kahneman's (2003) "fast thinking" processes. Cognitive theories of learning suggest that fluency processes are non-verbal and not necessarily willful or planned (Kellman and Massey 2013; Koedinger et al. 2012). Fluency processes are implicit processes involved in perceptual pattern recognition (Gibson 1969; Goldstone and Barsalou 1998; Kellman and Massey 2013; Richman et al. 1996). High efficiency in recognizing perceptual patterns results from perceptual chunking: relevant features serve to retrieve a corresponding schema from long-term memory that constitutes the relevant concepts (Richman et al. 1996; Taber 2013). The previously mentioned distinction between surface features and relevant features (Ainsworth 2006; Lowe 1993, 1994; Seufert 2003) plays an important role in perceptually fluency as well: learning perceptual patterns involves recognizing perceptual features that are meaningful (e.g., the number of sections in circle and rectangle representations) while disregarding surface features that are incidental (e.g., the color of circle and rectangle representations). Fluency processes use direct one-on-one mappings between perceptual chunks and concepts to discriminate and categorize GRs efficiently, without perceived mental effort (Koedinger et al. 2012; Richman et al. 1996). As a consequence, fluency processes allow students to automatically infer the conceptual information a given GR shows and to automatically integrate the information that multiple GRs show about a complex concept (Airey and Linder 2009).

Fluency processes are an important mechanism through which students learn domain knowledge from multiple GRs (Rau 2016; Goldstone and Barsalou 1998; Kellman et al. 2009; Richman et al. 1996). By automatically integrating information from multiple GRs, students can use the information provided by multiple GRs while having sufficient cognitive resources to use this information for higher-order reasoning about domain-relevant concepts (Goldstone and Barsalou 1998; Richman et al. 1996). Perceptual fluency is an important aspect of domain expertise (Dreyfus and Dreyfus 1986; Gibson 1969, 2000; Richman et al. 1996) and is viewed as an important goal in science education (Kozma and Russell 2005, Wu and Shah 2004), and in math education (Pape and Tchoshanov 2001).

Fluency-building activities are instructional activities designed to help students acquire fluency. The goal of these activities goes beyond merely enhancing perceptual fluency; the activities were designed so that they engage students in fluency processes that enhance their learning of domain knowledge. Given the argument above that fluency processes are an important mechanism of domain learning, engaging in fluency processes should enhance students' learning of domain knowledge. Further, fluency should accelerate students' learning of domain knowledge from subsequent instructional activities.

Although research on fluency-building activities is still relatively novel, prior research has yielded several instructional design principles for fluency-building activities (Gibson et al. 2010; Gibson 2000; Kellman and Massey 2013; Massey et al. 2011). First, fluency-building activities should ask students to *discriminate and categorize* numerous examples. Second, students should receive *immediate feedback* on these discrimination and classification activities (Massey et al. 2011). Third, students should practice with *many varied example GRs*, sequenced such that consecutive examples emphasize relevant perceptual features (Rau et al. 2014a; Kellman et al. 2009; Massey et al. 2011). Kellman et al. conducted several studies on fluency-building activities designed according to these principles for fractions learning, algebra learning, and chemistry learning (Kellman et al. 2008; Wise et al. 2000). Results from these studies show that students who received fluency-building activities designed according to these principles subsequently performed better not only on fluency tests but also on tests of domain knowledge, compared to students who did not receive fluency-building activities.

In sum, prior research suggests that fluency-building activities that engage students in fluency processes enhance their learning of domain knowledge.

Interplay among sense-making competency and perceptual fluency

A variety of literatures acknowledge that domain expertise involves sense-making competencies and perceptual fluency, including theories of cognitive skill acquisition (Anderson 1983; Koedinger et al. 2012; Ohlsson 2008; Richman et al. 1996), the STEM education literature (e.g., Kozma and Russell 2005; Patel and Dexter 2014), and educational practice guides (NCTM 2006; National Mathematics Advisory Panel, NMAP 2008; Siegler et al. 2010). These literatures seem to *implicitly* assume that the acquisition of sense-making competencies precedes the acquisition of fluency; and therefore that instruction should first provide sense-making activities and then fluency-building activities. We refer to this assumption as the *sense-making-first hypothesis*. Yet, recent research suggests an alternative hypothesis: the *fluency-first hypothesis*, which proposes that fluency can enhance the acquisition of sense-making competencies and therefore instruction should first provide fluency-building activities and then sense-making activities. Figure 2 illustrates the rationale underlying each hypothesis. In the following, we discuss each hypothesis in turn.

Sense-making-first hypothesis

Many educational practice guides seem to assume that students acquire sense-making competencies before fluency. Consequently, they suggest that students work on sense-making activities before they work on fluency-building activities. For example, the National Council of Teachers of Mathematics (NCTM 2006) expects sense-making of fractions representations by the end of grade 5. Fluency in efficiently interpreting and using fractions representations is expected at the end of grade 8.

In favor of the sense-making-first hypothesis, there is empirical evidence suggesting that students may not be able to acquire fluency unless they have prior sense-making competencies. One might argue that fluency-building activities assume that students have some

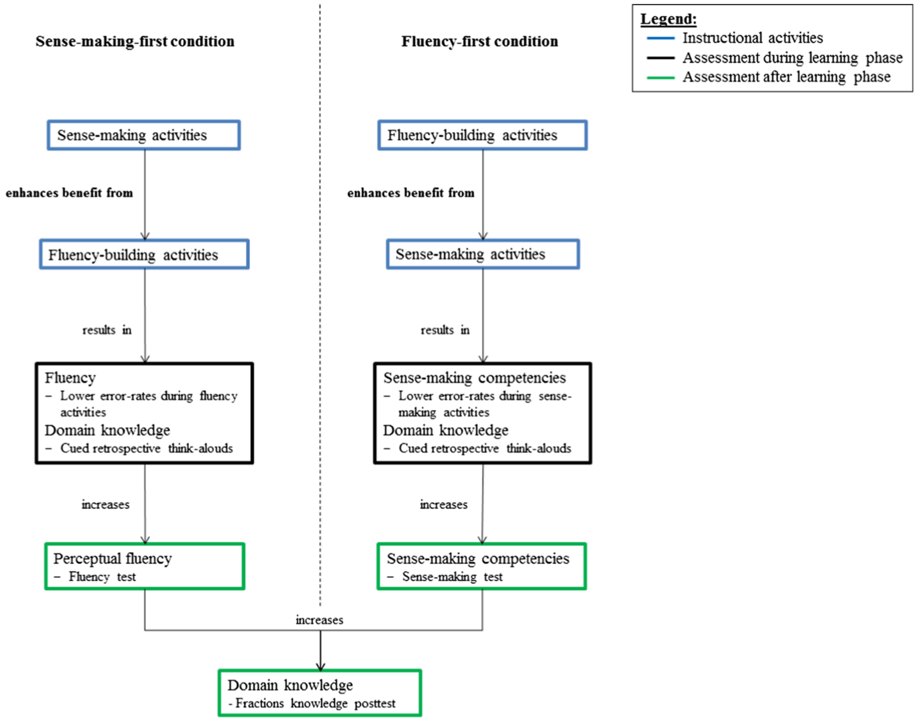


Fig. 2 Theoretical model of the sense-making-first hypothesis (*left*) and fluency-first hypothesis (*right*), detailing the mechanisms by which the sequence of sense-making activities and fluency-building activities (*blue*) results in learning outcomes (representational competencies and domain knowledge) during the learning phase (*black*) and after the learning phase (*green*). (Color figure online)

prior level of sense-making competencies; for example, that they know which perceptual features to attend to and which perceptual features of different GRs show the same concepts. If students do not have sense-making competencies, asking students to engage in fluency processes requires that they induce how different GRs map to one another (see Kellman et al. 2008, 2009). Yet, research shows that learning such mappings is a difficult task that students typically do not attempt spontaneously (Ainsworth et al. 2002; Rau et al. 2014a). If students do attempt to induce such mappings, they may employ inefficient learning strategies (e.g., trial and error), which might impede their benefit from fluency-building activities. Indeed, Kellman et al.' research on fluency-building activities seems to implicitly adopt the sense-making-first hypothesis: their participants were typically not novices but had considerable prior knowledge about the domain-relevant concepts, which likely involved sense-making competencies.

Fluency-first hypothesis

An alternative hypothesis is the *fluency-first hypothesis*, which suggests that fluency can enhance the acquisition of sense-making competencies. Even though the fluency-first hypothesis may seem counterintuitive at first glance (how can students fluently make connections among GRs if they cannot make sense of what the connections mean?), there are several theoretical accounts that lend credibility to the fluency-first hypothesis. As mentioned, prior research on fluency suggests that fluency allows students to efficiently integrate information from multiple GRs without requiring significant cognitive capacity, hence freeing cognitive resources for complex reasoning (Koedinger et al. 2012; Kellman et al. 2009). Because sense-making processes are complex reasoning processes that require considerable cognitive capacity, perceptual fluency may enhance sense-making processes. Empirical evidence for this notion comes from a few recent studies. First, recent mathematics cognition research provides evidence that students' fluency with GRs that depict fractions as magnitudes can promote their ability to make sense of fractions as parts of a whole (Matthews and Chesney 2015). Second, embodied cognition research shows that perceptual experiences with physical objects can enhance students' ability to fluently process information about abstract concepts. This fluency, in turn, enhances their ability to make sense of abstract concepts (Lindgren 2012; Nathan et al. 2014). Third, studies of socio-cultural learning processes show that students often become fluent with representations first (e.g., they often use representations and talk about them without fully understanding how they show information), which helps them acquire conceptual understanding of the representations (e.g., Airey and Linder 2009; Wertsch and Kazak 2011).

Own prior research

The sense-making-first and fluency-first hypotheses have, to the best of our knowledge, not been empirically tested in the context of learning with multiple representations because studies on sense-making activities and on fluency-building activities have been separate lines of research in the literature on learning with representations. Our own prior research (Rau et al. 2016) presents a first step towards integrating these two lines of research.

In a prior experiment on fractions learning, we investigated whether receiving sense-making activities and/or fluency-building activities enhances students' benefit from multiple GRs, compared to a control condition that did not receive either activity (Rau et al. 2016). Results showed an interaction effect such that students showed the highest learning gains on a fractions knowledge posttest if they received both sense-making and fluency-building activities. Receiving only sense-making activities or receiving only fluency-building activities was *less effective* than the control condition. These findings make two contributions. First, they provide evidence that supporting sense-making competencies and fluency enhances students' learning of domain knowledge. Second, the fact that *only* the combination of sense-making activities and fluency-building activities improved learning suggests that there is a more complex interplay among sense-making processes and fluency processes than anticipated. One possible mechanism is that sense-making competencies enhance students' ability to learn from fluency-building activities. This mechanism corresponds to the sense-making-first hypothesis. Another possible mechanism is that fluency enhances students' ability to learn from sense-making activities. This mechanism corresponds to the fluency-first hypothesis.

We tested these mechanisms with a causal path analysis using data on errors students made while they worked on sense-making activities and fluency-building activities (Rau et al. 2016). We found that students who had previously worked on sense-making activities showed higher performance when they worked on fluency-building activities than students who did not receive sense-making activities, which statistically mediated the advantage of this condition on learning outcomes. This finding suggests that sense-making competencies enhance students' learning from fluency-building activities. Hence, our prior findings are in line with the *sense-making-first hypothesis*.

One critical limitation of our prior experiment was that we implicitly adopted the sense-making-first hypothesis—which, as argued above, seems to be a prevalent assumption in education. Consequently, we had provided sense-making activities before fluency-building activities (for each topic). Therefore, the question remains open whether providing sense-making activities before fluency-building activities is more effective than providing fluency-building activities before sense-making activities. The present experiment addresses this question.

Hypotheses and predictions

Even though prior research has not yet explicitly tested the sense-making-first hypothesis or the fluency-first hypothesis, theoretical accounts exist in favor of either hypothesis. Both hypotheses make specific predictions that we can test empirically. We consider predictions for (1) a sequence that provides sense-making activities before fluency-building activities (sense-making-first condition) versus (2) a sequence that provides fluency-building activities before sense-making activities (fluency-first condition). Table 1 provides an overview of the predictions each hypothesis makes for the dependent measures we consider.

Regarding *representational competencies*, the sense-making-first hypothesis predicts that sense-making competencies enhance students' benefit from fluency-building activities. Consequently, fluency-building activities should be easier for students in the sense-making-first condition. Therefore, they should make fewer errors on fluency-building activities during the learning phase (prediction 1a) and show higher gains on a fluency test immediately after fluency-building activities (prediction 1b). Regarding *domain knowledge*, the sense-making hypothesis proposes that sense-making activities should enhance students' ability to engage in conceptual reasoning about fractions during the learning phase. Therefore, it predicts that students in the sense-making-first condition engage in more conceptual reasoning about fractions (prediction 3a). Consequently, students in the sense-

Table 1 Predictions made by the sense-making-first hypothesis and fluency-first hypothesis

Prediction	Measure and expected effect	Results
Sense-making-first hypothesis	In sense-making-first condition	
Effects on representational competencies		
Prediction 1a	Fewer errors on fluency activities during learning phase	Partially confirmed
Prediction 2a	Higher fluency after fluency-building activities	Confirmed
Effects on domain knowledge		
Prediction 3a	Higher conceptual reasoning about fractions during learning phase	Confirmed
Prediction 4a	Higher fractions-knowledge after learning phase	Partially confirmed
Mediation effects		
Prediction 5a	Fewer errors on fluency activities mediate higher fractions-knowledge	Partially confirmed
Fluency-first hypothesis	In fluency-first condition	
Effects on representational competencies		
Prediction 1b	Fewer errors on sense-making activities during learning phase	Contradicted
Prediction 2b	Higher sense-making competencies after sense-making activities	Not confirmed
Effects on domain knowledge		
Prediction 3b	Higher conceptual reasoning about fractions during learning phase	Contradicted
Prediction 4b	Higher fractions-knowledge after learning phase	Contradicted
Mediation effects		
Prediction 5b	Fewer errors on sense-making activities mediate higher fractions-knowledge	Contradicted

making-first condition will show higher learning gains (prediction 4a). Finally, the sense-making first hypothesis predicts a *mediation effect.* The sense-making hypothesis proposes that students in the sense-making condition have higher learning outcomes on the fractions knowledge test (prediction 4a) *because* sense-making competencies enhance their benefit from sense-making activities, indicated by fewer errors on fluency-building activities (prediction 1a). Hence, a reduction of errors on fluency-building activities should mediate the advantage of the sense-making-first condition on the fractions knowledge test (prediction 5a).

Regarding *representational competencies*, the fluency-first hypothesis predicts that fluency enhances students' benefit from sense-making activities. Consequently, sense-making activities should be easier for students in the fluency-first condition, and therefore they should make fewer errors on sense-making activities during the learning phase (prediction 2a). Further, students in the fluency-first condition should show higher gains on a sense-making test after sense-making activities (prediction 2b). Regarding *domain knowledge*, the fluency-first hypothesis proposes that fluency processes are crucial mechanisms for learning of domain knowledge. Therefore, students in the fluency-first condition should engage in more conceptual reasoning about fractions (prediction 3b), which should result in higher learning gains on the fractions knowledge test (prediction 4b). Finally, the fluency-first hypothesis predicts a *mediation effect* based on the rationale that students in the fluency-first condition have higher learning outcomes on the fractions knowledge test (prediction 4b) *because* fluency enhances their benefit from sense-making activities, indicated by fewer errors on sense-making activities (prediction 1b). Hence, a reduction of error rates on sense-making activities should mediate the advantage of the fluency-first condition on the fractions knowledge test (prediction 5b).

Methods

To test these hypotheses, we conducted a lab experiment that compared a condition that received sense-making activities before fluency-building activities (sense-making-first condition) to a condition that received fluency-building activities before sense-making-activities (fluency-first condition), as detailed in the following.

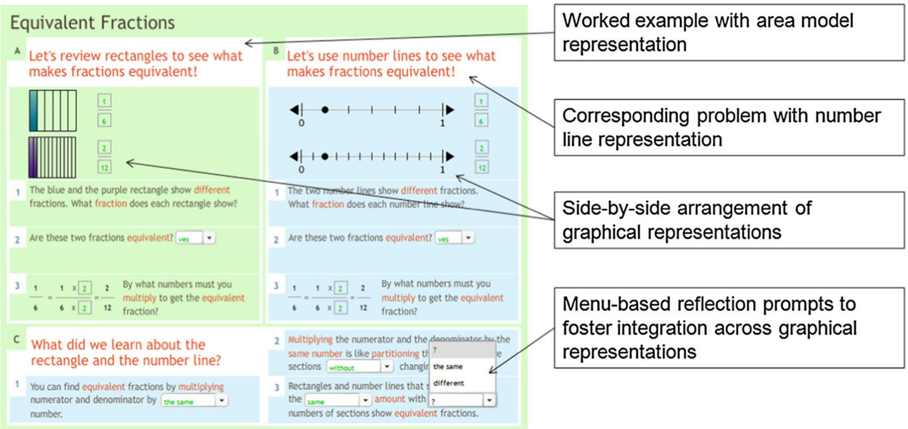
Participants

Seventy-four 3–5th-grade students from western Pennsylvania participated in the experiment. Students were recruited through local advertisements.

Materials

We conducted the experiment in the context of the Fractions Tutor, an intelligent tutoring system (ITS) for elementary-school level fractions learning that we created to investigate the sense-making-first and fluency-first hypotheses. ITSs are a type of educational technology that pose complex problem-solving activities and provide individualized step-by-step guidance at any point during the problem-solving process (VanLehn 2011). At the heart of ITSs lies a cognitive model of the students' problem-solving knowledge. This model allows ITSs to detect multiple strategies a student might use to solve a problem, and to provide detailed feedback and on-demand hints on how to solve the next step. Traditional ITSs use a rule-based cognitive model (Aleven 2010) that is based on production-rule theories of learning, such as ACT-R (Anderson et al. 1995). The Fractions Tutor is a newer type of ITS, called example-tracing tutors (Aleven 2010; Aleven et al. 2009; Aleven et al. 2016), which provide the same tutoring behaviors as other ITSs, but instead of a rule-based cognitive model, rely on generalized examples of correct and incorrect solution paths. The Fractions Tutor is based on results from prior classroom experiments and user-centered design research (Rau et al. 2013). It provides a variety of interactive GRs, shown in Fig. 1. For this experiment, we selected activities from the units on fractions equivalence and fractions comparison that were designed to support sense-making processes and fluency processes.

The Fraction Tutor's *sense-making activities* were designed based on the principles described above (see "Two types of representational competencies" section), as illustrated in Fig. 3. First, to help students to make *active comparisons* among GRs, the Fractions Tutor uses worked examples (Renkl 2005). Students are first presented with a worked example (Fig. 3a) that uses one of the area models (i.e., circle or rectangle) to demonstrate how to solve a fractions problem. Students complete the last step of the worked-example problem (step A.3) and receive feedback from the tutor (e.g., correct steps are shown with



Equivalent Fractions

A Let's review rectangles to see what makes fractions equivalent!

1 The blue and the purple rectangle show different fractions. What fraction does each rectangle show?

2 Are these two fractions equivalent? yes

3 $\frac{1}{6} = \frac{1 \times \boxed{2}}{6 \times \boxed{2}} = \frac{2}{12}$ By what numbers must you multiply to get the equivalent fraction?

B Let's use number lines to see what makes fractions equivalent!

1 The two number lines show different fractions. What fraction does each number line show?

2 Are these two fractions equivalent? yes

3 $\frac{1}{6} = \frac{1 \times \boxed{2}}{6 \times \boxed{2}} = \frac{2}{12}$ By what numbers must you multiply to get the equivalent fraction?

C What did we learn about the rectangle and the number line?

1 You can find equivalent fractions by multiplying numerator and denominator by the same number.

2 Multiplying the numerator and the denominator by the same number is like partitioning th e sections without changir

3 Rectangles and number lines that s the same amount with ? numbers of sections show equivalent fractions.

Worked example with area model representation

Corresponding problem with number line representation

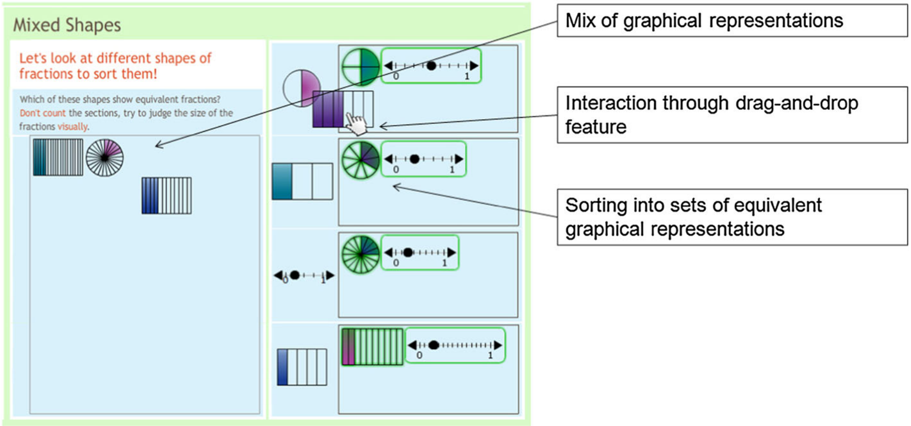
Side-by-side arrangement of graphical representations

Menu-based reflection prompts to foster integration across graphical representations

Fig. 3 Screen shot of sense-making activity in the equivalent fractions unit

green font). With the worked example still on the screen, they are then presented with an analogous problem in which they have to use the numberline (Fig. 3b). Students complete this analogous problem with help (i.e., step-level hints and feedback) from the tutor. They are prompted to use the area model to help them complete the numberline problem, so as to encourage them to establish mappings between corresponding perceptual features (e.g., the sections between 0 and the dot in the numberline correspond to the shaded sections in the circle because both perceptual features show the numerator). Second, students are prompted to *self-explain* these mappings. To this end, students receive self-explanation prompts at the end of each problem (section C in tutor screen shown in Fig. 3). The prompts ask students to relate the two GRs by reasoning about how they depict fractions. Self-explanation prompts were implemented in a fill-in-the blank format with drop-down menus. Similar simple formats have been shown to be effective in prior research with ITSs or other educational technologies (Aleven and Koedinger 2002; Atkinson et al. 2003; Rau et al. 2016) and more effective than open-ended forms of self-explanation prompts (Gadgil et al. 2012; Johnson and Mayer 2010; van der Meij and de Jong 2011). Finally, students receive *assistance* in the form of feedback on their problem-solving activities with the numberline and on their responses to self-explanation prompts. Hints and error feedback messages were specifically designed to help students focus on conceptually relevant features while making comparisons and while self-explaining connections among the GRs.

The *fluency-building activities* are similar to Kellman et al.'s (2008, 2009) interventions, as illustrated in Fig. 4. First, students are asked to *discriminate and categorize* GRs: in Fig. 4, they have to sort GRs (provided in the box on the left) into sets of equivalent fractions (represented by the boxes on the right), using a drag-and-drop feature (i.e., each representation in the box on the left can be picked up with the mouse and dragged over and dropped in the appropriate box on the right). Second, students receive *immediate feedback* on this task (e.g., a green halo indicates that the representation has been dropped in the appropriate box). Third, the fluency-building activities provide practice opportunities with *many varied GRs*. To encourage students' engagement in non-verbal processes, the problem instructions encourage them to solve fluency-building activities visually by estimating the relative size of the fractions, rather than by solving it conceptually or computationally.



Mixed Shapes

Let's look at different shapes of fractions to sort them!

Which of these shapes show equivalent fractions? Don't count the sections, try to judge the size of the fractions visually.

Mix of graphical representations

Interaction through drag-and-drop feature

Sorting into sets of equivalent graphical representations

Fig. 4 Screen shot of a fluency activity in the equivalent fractions unit

Assessments

In the following, we describe how we assessed learning outcomes and learning processes that correspond to the predictions summarized in Table 1.[1]

Assessments of representational competencies

Error-rate measures To assess representational competencies during the learning phase, we considered error rates during fluency-building activities and during sense-making activities. To this end, we used the log data from the Fractions Tutor. The logs provide a detailed record of students' interactions with the Fractions Tutor at the "transaction" level (i.e., attempts at steps, hint requests, etc.). Following standard practice in ITS research (Koedinger et al. 2010), we computed error rates as the number of errors students made per step. We computed four error-rate measures: error rates on sense-making activities in the equivalence unit, error rates on sense-making activities in the comparison unit, error rates on fluency-building activities in the equivalence unit, and error rates on fluency-building activities in the comparison unit.

Sense-making and fluency tests To assess representational competencies after students worked on sense-making and fluency-building activities (see detailed description of the procedure below), we assessed students' sense-making competencies with a *sense-making test* and their fluency in making connections with a *fluency test*. We created two versions for each test; a 4-item test form for the equivalence unit, and a 4-item test forms for the comparison unit of the Fractions Tutor. For each test version, we created two different test forms to be used at as an intermediate test and a posttest (see procedure below). The different test forms included structurally identical items that used different numbers. The sequence of the two test forms was counterbalanced. Sample test items for are provided in the Online Appendix.

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[1] In addition to the assessments detailed below, we assessed eye-tracking data. Because the eye-tracking data did not yield results relevant to the research questions we investigate in this article, we do not report eye-tracking data. Results from the analysis of eye-tracking data are reported in Rau et al. (2014b).

Assessments of domain knowledge

Conceptual reasoning about fractions To assess students' reasoning about fractions concepts during the learning phase, we conducted cued retrospective think-alouds with all students.[2] We used the cued retrospective think-aloud method described by Van Gog et al. (2005), which has been compared against concurrent reporting methods and has been shown to yield valid information about students' reasoning processes during problem solving. In our study, we randomly selected four activities from the Fractions Tutor: one equivalence sense-making activity, one equivalence fluency activity, one comparison sense-making activity, and one comparison fluency activity (see Table 2). The experimenter asked predefined questions about how the student solved each step (e.g., "In this step [points at the step], you immediately selected 'yes' from the menu, that the fractions are equivalent. How did you solve that step?"). We coded utterances from the cued retrospective think-alouds using a coding scheme from our prior research (Rau et al. 2014b). Specifically, we counted each utterance as an instance of conceptual reasoning if the student explained a fractions concept correctly (e.g., "when there are three out of five and three out of seven, the three out of five is larger because the parts are bigger"). Interrater reliability between two independent coders of 10% of the data was substantial with $\kappa = 0.66$ (Landis and Koch 1977).

Fractions-knowledge tests To assess domain knowledge after the learning phase (i.e., predictions 4a/b), the students completed *fractions knowledge tests*. The fractions knowledge tests included transfer items that were structurally different from the problems covered in the Fractions Tutor and had nine items. We created two different test forms to be used at a pretest and a posttest. The different test forms included structurally identical items that used different numbers. The sequence of the two test forms was counterbalanced. Sample test items are provided in the Online Appendix.

Experimental design and procedure

Figure 5 provides details about the sequence and number of instructional and assessment activities for each experimental condition.

Students first completed a fractions knowledge pretest. They then worked on the Fractions Tutor. Students were randomly assigned to one of two versions of the Fractions Tutor that differed only in the order in which students received sense-making activities and

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[2] The procedure for the cued retrospective think-alouds changed midway during the experiment. The procedure change only affected the cued retrospective think-alouds (no other aspects of the experimental procedure, because the cued retrospective think-alouds came last), and it equally affected both experimental conditions. The change was necessary due to delayed arrival of eye-tracking equipment. Specifically, the first 38 (of 74) students were asked to do a retrospective think-aloud using video recordings without eye-gaze recordings. For the remaining 36 students, eye-tracking data were recorded with an unobtrusive remote eye-tracker. (Specifically, we used an SMI RED 250—which uses an infrared camera attached under a computer monitor to record eye-gaze behaviors. The interactions with the computer were no different than without the eye-tracker.) For the cued retrospective think-alouds for these 36 students, we used eye-gaze recordings as cues, following the method proposed by Van Gog et al. (2005). For each activity, the experimenter played back the recorded eye-gaze behaviors. The eye-gaze recordings depict the student's eye-gaze as a circle, overlaid with a background-screen recording showing the student's interactions with the problem-solving activity. In replaying the eye-gaze recording, the experimenter first explained what the eye-gaze circle shows, and then paused after each step for a think-aloud prompt. The remainder of the cued retrospective think-alouds proceeded as for the first 38 students.

Table 2 Means and standard deviations (in parentheses) on learning outcome measures by test-time and experimental condition

Test	Test-time	Sense-making-first condition	Fluency-first condition
Sense-making tests (equivalence and comparison)	Intermediate tests	0.49 (0.36)	0.39 (0.38)
	Posttests	0.41 (0.33)	0.45 (0.41)
Fluency tests (equivalence and comparison)	Intermediate tests	0.62 (0.35)	0.60 (0.32)
	Posttests	0.65 (0.34)	0.50 (0.31)
Fractions-knowledge tests	Pretest	0.45 (0.35)	0.53 (0.34)
	Posttest	0.51 (0.36)	0.47 (0.32)

fluency-building activities. Students in the *sense-making-first condition* received sense-making activities before fluency-building activities. By contrast, students in the *fluency-first condition* received fluency-building activities before sense-making activities. This procedure was implemented for both Fractions Tutor units, namely equivalent fractions and fraction comparison. After solving the first half of the activities in the given unit, students completed the intermediate sense-making test and the intermediate fluency test. After completing the remaining tutor activities for the given unit, students completed the sense-making posttest and fluency posttest. At the end, students completed the fractions knowledge posttest.

Causal path analysis

To test predicted mediation effects, we used causal path analysis modeling. The sense-making-first condition predicts that a reduction of error rates on fluency-building activities during the learning phase mediates the advantage of the sense-making-first condition on the fractions knowledge test (prediction 5a), whereas the fluency-first hypothesis predicts that a reduction of error rates on sense-making activities mediates the advantage of the fluency-first condition on the fractions knowledge test (prediction 5b). We tested two models; one model to test the sense-making-first hypothesis that focuses on error rates during fluency-building activities, and a second model to test the fluency-first hypothesis that focuses on error rates during sense-making activities.

Causal path analysis modeling yields statistical models of mediation effects that (1) are theoretically plausible, (2) fit the data well, and (3) contain only edges that describe statistically reliable effects. To construct the causal path analysis models, we used an automatic algorithm that searches for models that are theoretically plausible and consistent with the data (Chickering 2002; Spirtes et al. 2000). To do so, we used the Tetrad IV program's[3] GES algorithm.

A first step in the analysis is to select variables to include in the model. The *independent variable* in the causal path analysis was condition (i.e., sense-making-first condition versus fluency-first condition). The *dependent variables of interest* were students' pretest and

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[3] Tetrad, freely available at www.phil.cmu.edu/projects/tetrad, contains a causal model simulator, estimator, and over 20 model search algorithms, many of which are described and proved asymptotically reliable in Spirtes et al. (2000).

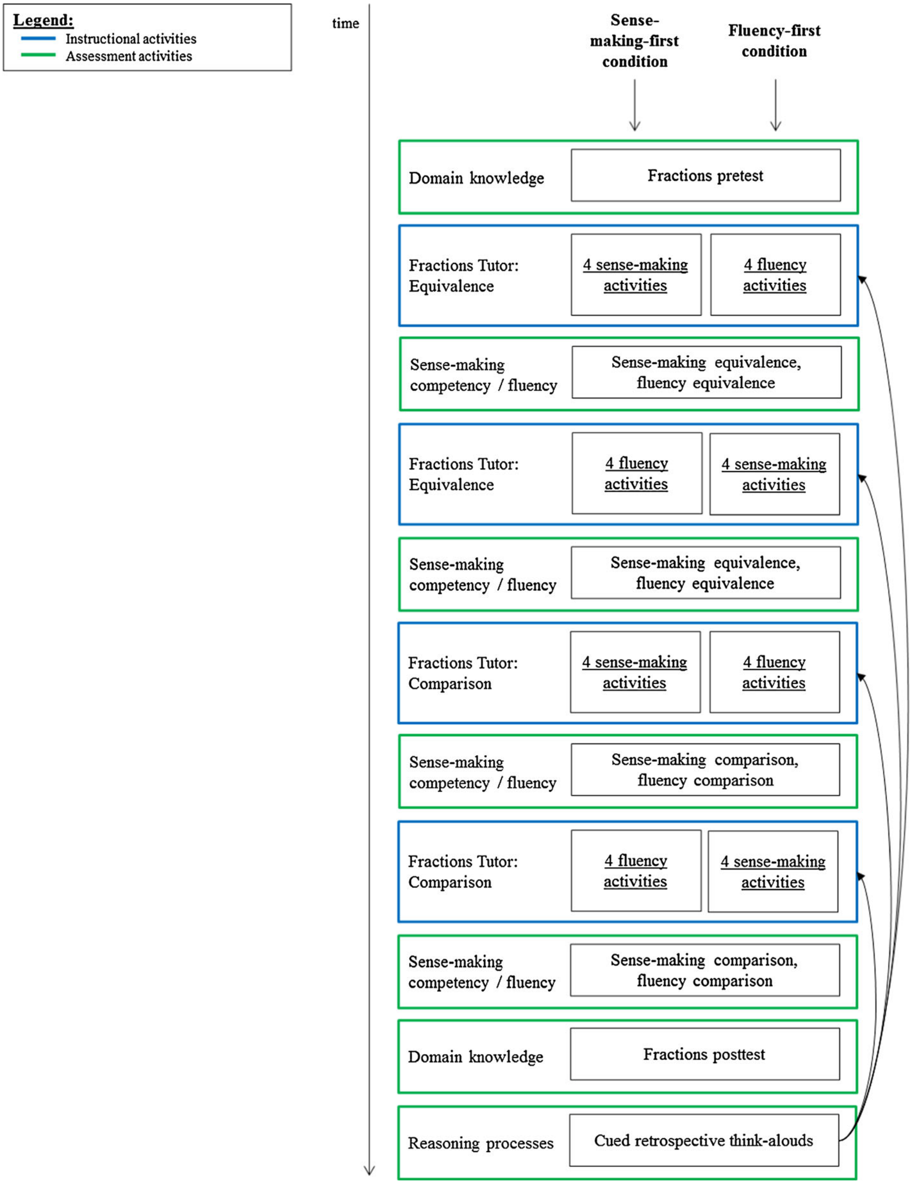

Fig. 5 Sequence of instructional and assessment activities by experimental condition

posttest scores on the fractions knowledge test. The *mediators* were error rates on equivalence sense-making activities, equivalence fluency-building activities, comparison sense-making activities, and comparison fluency-building activities.

The second step is to specify assumptions about the model. When conducting a model search, we can narrow the search space based on the knowledge we have about the nature of our data. We assumed that our condition variable is exogenous and causally independent, that the pretest is not influenced by condition, that the pretest is an exogenous variable and causally independent of condition, that performance on the posttest cannot influence the mediators, and that the mediators (i.e., error rates) can influence performance on the posttest.

The third step is to specify the fully saturated model. The fully saturated model for each hypothesis contains all possible edges (or "effects") compatible with the experimental design. Figure 6 illustrates that, even if we consider only two mediators for each model, there are over 1000 (2^{10}) distinct models consistent with our background knowledge and that are plausible tests for our mediation hypotheses.

The fourth step is to let the GES algorithm search for the best-fitting model within the search space. The outcomes of the model search are two causal path analysis models, one testing the fluency hypothesis, one testing the sense-making hypothesis, each consistent with the data and hence allowing us to trust the parameters of the model.

The final step is to examine whether the models support the mediation hypotheses. In general, evidence for a full mediation effect corresponds to a causal path model that shows a significant effect of condition on error rates, a significant effect of error rates on learning outcomes, but no significant effect of condition on learning outcomes (whereas without a mediator, there is a significant effect of condition on learning outcomes). Evidence for a partial mediation effect corresponds to a causal path model that shows a significant effect of condition on error rates, a significant effect of error rates on learning outcomes, and also a significant effect of condition on learning outcomes.

Results

Five students were excluded from the analysis because they did not complete both units of the Fractions Tutor or because they were statistical outliers at the fractions knowledge pretest. Thus, we report data from $N = 69$ students ($n = 37$ in the sense-making-first condition, $n = 32$ in the fluency-first condition). Table 2 provides a summary of students' scores on each test. In the following, we present the results related to each of the predictions in Table 1. The right-most column of Table 1 summarizes whether the results support the hypotheses.

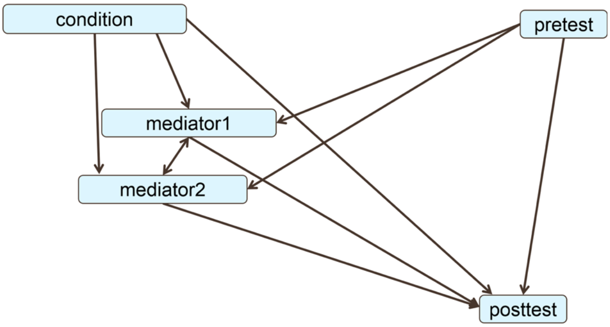

Fig. 6 Search space of causal path analysis models compatible with the experimental design

Effects on representational competencies

To test effects on error rates during the learning phase (see Table 1, predictions 1a and 1b), we computed MANCOVAs with pretest performance as covariate, and error rates on sense-making activities and fluency-building activities for the equivalence and comparison units, respectively. The sense-making-first hypothesis predicts that the sense-making-first condition will have lower error rates on fluency-building activities (prediction 1a). The effect of condition was not significant for equivalence-sense error rates, $F(1, 66) = 2.04$, $p > 0.10$, but it was significant for comparison-sense error rates, $F(1, 66) = 2.80$, $p < 0.10$, $\eta_p^2 = 0.04$, such that the sense-making-first condition showed lower error rates. These findings partially confirm prediction 1a.

The fluency-first hypothesis predicts that the fluency-first condition will have lower error rates on sense-making activities (prediction 1b). The effect of condition was marginally significant for equivalence-fluency error rates, $F(1, 66) = 2.79$, $p = 0.10$, $\eta_p^2 = 0.04$, but in the opposite direction: the sense-making-first condition showed lower error rates on sense-making activities during the learning phase than the fluency-first condition. There were no significant differences on comparison-fluency error rates, $F(1, 66) = 2.61$, $p > 0.10$. These findings contradict prediction 1b.

To test prediction 2a, that the sense-making-first condition will show higher learning outcomes on the *fluency tests* (administered after set of sense-making/fluency-building activities; see Fig. 5), we computed a repeated measures ANCOVA with condition as independent factor, test-time (fluency intermediate test and fluency posttest) as repeated factor, fractions knowledge pretest score and time spent on the Fractions Tutor as covariates. The dependent measure was students' scores on the fluency tests collapsed across the equivalence and comparison units. There was a marginally significant main effect of condition, $F(1,65) = 3.34$, $p < 0.10$, $\eta_p^2 = 0.05$, but no main effect of test time ($F < 1$) nor an interaction of test time with condition, $F(1,65) = 1.42$, $p > 0.10$. Posthoc comparisons revealed no significant differences between conditions on the fluency test at the first test time (i.e., when the fluency-first condition had received fluency-building activities but the sense-making-first condition had not) ($F < 1$). Posthoc comparisons showed a significant advantage of the sense-making-first condition over the fluency-first condition on the fluency test at the second test time (i.e., when both conditions had received fluency-building activities), $F(1,65) = 4.52$, $p < 0.05$, $\eta_p^2 = 0.07$. This result confirms prediction 2a.

To test prediction 2b, that the fluency-first condition will show higher learning outcomes on the *sense-making test*, we computed a repeated measures ANCOVA. There were no significant effects (Fs < 1). This finding does not confirm prediction 2b.

Effects on domain knowledge

We look at effects on students' fractions knowledge during and after the learning phase. First we look at conceptual reasoning during the learning phase, as assessed by cued retrospective think-alouds (see Van Gog et al. 2005). The sense-making-first hypothesis predicts that students in the sense-making-first condition engage in more conceptual reasoning about fractions (prediction 3a), whereas the fluency-first hypothesis predicts the opposite (prediction 3b). To test these predictions, we computed a χ^2 test on the number of conceptual reasoning utterances. The results show a significant difference between conditions, $\chi^2(1, N = 550) = 6.55$, $p < 0.05$, such that students the sense-making-first condition made significantly more conceptual reasoning utterances ($M = 8.24$ utterances per student) than students in the fluency-first condition ($M = 7.66$ utterances per student). This finding partially (i.e., marginally) confirms prediction 3a but contradicts prediction 3b.

Next, we look at effects on fractions knowledge after the learning phase, as measured by the fractions knowledge pretest and posttest (see Fig. 5). The sense-making-first hypothesis predicts that students in the sense-making-first condition will show higher learning gains on the fractions knowledge posttest (prediction 4a), whereas the fluency-first hypothesis predicts the opposite (prediction 4b). To test these predictions, we computed a repeated measures ANCOVA on the fractions knowledge test. Results showed a marginally significant interaction of test time with condition, $F(1,66) = 3.76$, $p < 0.10$, $\eta_p^2 = 0.05$, but no significant main effects of condition or test (Fs < 1). A posthoc comparison on the posttest with time spent on the Fractions Tutor and pretest as covariates showed a marginally significant advantage of the sense-making-first condition on the fractions knowledge posttest, $F(1,65) = 3.05$, $p < 0.10$, $\eta_p^2 = 0.05$. This finding partially (i.e., marginally) confirms prediction 4a but contradicts prediction 4b.

Mediation effects

Finally, we look at mediation effects detected through causal path modeling. First, we examined the causal path model that tests the prediction 5a (refer to "Causal path analysis" section). This model tested whether *error rates on fluency-building activities* mediate the advantage of the sense-making-first condition on the fractions knowledge posttest. Figure 7 shows the best model found by the GES algorithm. The model fits the data well, ($\chi^2 = 4.58$, df $= 4$, $p = 0.33$). Error rates on equivalence-fluency problems fully mediate a positive effect of the sense-making-first condition on the fractions knowledge posttest: students in the sense-making-first condition show lower error rates on equivalence-fluency activates, which statistically explains the advantage of the sense-making-first condition over the fluency-first condition on the fractions knowledge posttest. This finding confirms prediction 5a.

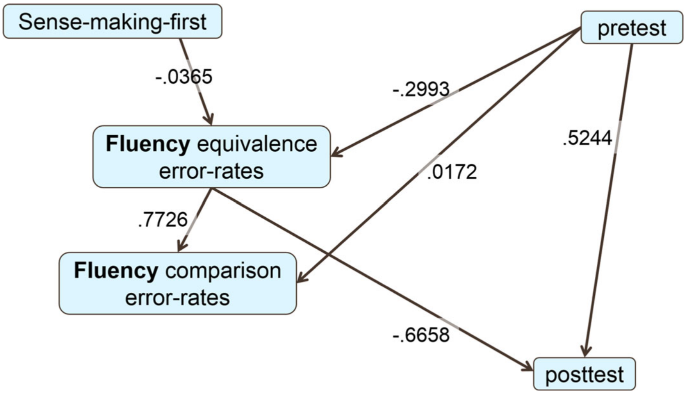

Fig. 7 The model found by the GES algorithm for the mediation hypothesis corresponding to the sense-making-first hypothesis: effect of the sense-making-first condition on the fractions-knowledge posttest through error-rates on fluency activities. Values are unstandardized coefficients

Next, we examined the causal path model that tests the prediction 5b (see "Causal path analysis" section). This model tested whether error rates *on sense-making activities* mediate the effect of condition on the fractions knowledge posttest. Figure 8 shows the best model found by the GES algorithm. The model fits the data well, ($\chi^2 = 3.38$, df $= 3$, $p = 0.38$). Error rates on sense-making activities fully mediate a negative effect of the fluency-first condition on the fractions knowledge posttest: students in the fluency-first condition show higher error rates on sense-making activities, which statistically explains their lower scores on the fractions knowledge posttest. This finding contradicts prediction 5b.

Discussion

Our research investigates whether and how helping students acquire representational competencies can help them achieve greater learning gains in domain knowledge. We investigated whether sense-making enhances perceptual fluency or vice versa. We built on our prior research, which showed that both sense-making competencies and perceptual fluency are important representational competencies that affect students' learning of domain knowledge (Rau et al. 2016). Our prior research led to open question about the how sense-making competencies and perceptual fluency interact. Our present experiment tested a commonly held assumption, namely that students may need to acquire sense-making competencies before perceptual fluency (sense-making-first hypothesis; see Fig. 2, left). We contrasted this assumption to an alternative hypothesis, which proposes that perceptual fluency can help students acquire sense-making competencies (fluency-first hypothesis; see Fig. 2, right). We tested a number of specific predictions made by the sense-making-first hypothesis and the fluency-first hypothesis, summarized in Table 1.

Summary of findings

First, let use review whether our results support the predictions made by the sense-making-first hypothesis (see Table 1, top; Fig. 2, left). With respect to *representational*

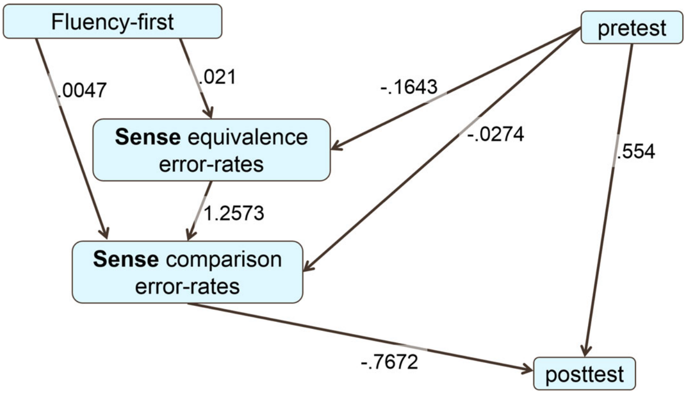

Fig. 8 The model found by GES for the mediation hypothesis of the effect of the understanding-first condition on the transfer posttest through error rates on sense-making problems. Values are unstandardized coefficients

competencies, results showed that students in the sense-making-first condition make marginally fewer errors on fluency-building activities during the learning phase, which partially confirms prediction 1a. They also showed higher learning gains on a fluency test that was given for each learning phase, confirming prediction 2a. With respect to *domain knowledge*, results from the cued retrospective think-alouds showed that students in the sense-making-first condition engaged in significantly more conceptual reasoning about fractions during the learning phase, confirming prediction 3a. Further, students in the sense-making-first condition showed marginally higher learning gains on a fractions-knowledge posttest after the learning phase, which partially confirms prediction 4a. Finally, the causal path analysis showed that the sense-making-first condition's lower error rates on fluency-building activities *mediated* its advantage of the sense-making-first condition on the fractions knowledge posttest, confirming prediction 5a.

Second, let us review whether our results support the predictions made by the fluency-first hypothesis (see Table 1, bottom; Fig. 2, right). With respect to *representational competencies,* we found that students in the fluency-first condition made marginally *more* errors on sense-making activities during the learning phase, which contradicts prediction 1b. We found no differences between conditions on the sense-making posttest, which does not confirm prediction 2b. With respect to *domain knowledge*, the cued retrospective think-alouds showed that students in the sense-making-first condition engaged in significantly more conceptual reasoning about fractions, which contradicts prediction 3b. Further, students in the sense-making-first condition showed marginally higher learning gains on the fractions-knowledge posttest, which contradicts prediction 4b. Finally, the causal path analysis showed that the increase of students' errors on sense-making activities in the fluency-first condition statistically *mediated* the marginal disadvantage of the fluency-first condition on the fractions knowledge posttest, which contradicts the prediction 5b.

In sum, our results provide full or partial support for each of the predictions made by the sense-making-first hypothesis. By contrast, our results stand in contrast to all predictions made by the fluency-first hypothesis.

Interactions among sense-making activities and fluency-building activities

Why might sense-making activities enhance students' benefit from subsequent fluency-building activities? Results from the causal path analysis provide insights into the nature of this relationship. Students who start with sense-making activities make fewer errors on fluency-building activities. This finding indicates that sense-making activities seem to prepare students' learning from fluency-building activities. Sense-making competencies may be necessary for students' benefit from fluency-building activities because they equip students with the knowledge necessary to identify conceptually relevant perceptual features. Without sense-making competencies, students may have to induce which perceptual features are relevant when asked to fluently make connections among multiple GRs; and they may (at best) be inefficient at accomplishing this task or (at worst) infer incorrect mappings.

Why might working on fluency-building activities first reduce students' ability to learn from subsequent sense-making activities? This finding came somewhat as a surprise because it was not predicted by the sense-making-first hypothesis while contradicting the fluency-first hypothesis, which states that fluency would enhance rather than impede students' acquisition of sense-making competencies. Hence, our results seem to extend the sense-making-first hypothesis by indicating that there are potential *costs* associated of acquiring perceptual fluency without having the prerequisite sense-making competencies.

It may be that working on fluency-building activities "primes" students to rely on perceptual features rather than to make sense of connections, perhaps creating an "illusion of knowing" (Glenberg et al. 1982) that makes them "careless" as they solve sense-making activities. This interpretation is in line with the concern that students who are overly influenced by the perceptual features of a representation may not pay attention to the conceptually relevant aspects of a representation, even though this is crucial to their learning of domain knowledge (Bieda and Nathan 2009). Providing students with fluency-building activities too early might encourage superficial strategies that impede learning from sense-making activities. The process by which students become perceptually fluent might be perceptual chunking: rather than perceiving each feature of a GR (e.g., colored sections in a circle show the numerator and total sections show the denominator), students may treat the entire GR as a single perceptual chunk (e.g., the proportion of the area in a circle that is colored). The acquisition of perceptual chunks (through fluency processes) might hinder students' acquisition of fine-grained chunks in sense-making activities. Having a perceptual chunk might allow the student to "bypass" learning of which feature in a given GR depicts a certain concept. For example, a student might rely on the proportion shown in a circle representation, without learning how numerator and denominator are depicted. Because both sense-making competencies and fluency are important aspects of learning the domain knowledge (Rau et al. 2016b), students who can "bypass" sense-making processes are at a disadvantage.

Contributions to prior research

It is important to note that our findings leave some room for doubt. We found differences on error rates only for some of the Fractions Tutor units. Furthermore, we found only a marginally significant difference between conditions on the fractions knowledge posttest. This may be due to the relatively short intervention with only two units of the Fractions Tutor, lasting about 1 h. Further research, possibly with a more comprehensive intervention, is necessary to support our thus far tentative conclusions.

Yet, our research makes several contributions to the extant literature on learning with multiple representations. To the best of our knowledge, the sense-making-first and fluency-first hypotheses have not been empirically tested in the context of learning with multiple representations because prior research has focused either only on sense-making activities or only on fluency-building activities. Given that our findings support the sense-making-first hypothesis (see Fig. 2, left), they support the commonly held assumption that students acquire sense-making competencies before fluency. At a practical level, this assumption implies that instruction should provide sense-making activities before fluency-building activities. Our results support this practice.

This finding extends prior research that has focused only on sense-making competencies, without taking perceptual fluency into account (e.g., Ainsworth 2006; Bodemer and Faust 2006; Schnotz 2005, 2014; Seufert 2003). Our findings suggest that sense-making activities not only support students' learning of domain knowledge, but also enhance students' ability to acquire other representational competencies that also play an important role in students' learning of domain knowledge. Hence, interventions for coherence formation (Seufert 2003), interventions that emphasize correspondences and complementary functions of multiple representations (Ainsworth 2006), and interventions that help students integrate information from different representations (Bodemer and Faust 2006; Schnotz 2005, 2014) may not only enhance students' acquisition of sense-making competencies and domain knowledge, but they may also enhance students' learning of perceptual fluency.

Our findings also extend prior research on perceptual fluency (e.g., Kellman et al. 2009), which had not yet tested whether sense-making competencies are a prerequisite for students' benefit from fluency-building activities. Our results suggest potential boundary conditions for the effectiveness of fluency-building activities; they may only help students learn domain knowledge if students have prerequisite sense-making competencies. Hence, Kellman's fluency-building activities may have been only effective because students had received prior instruction on sense-making competencies. Finally, the fact that we implemented support for sense-making processes and fluency processes in learning activities in which students solved domain-relevant problems contributes to research on the *representation dilemma*. The representation dilemma describes the conundrum that students have to learn about GRs (e.g., how to make connections among GRs) concurrently with learning domain knowledge from the GRs. Our results illustrate that carefully designed support for representational competencies can be overlaid with learning activities that focus on domain knowledge. This type of support helps students acquire representational competencies while they acquire domain knowledge.

Conclusion

Overall, our results support several of the specific predictions made by the sense-making-first hypothesis. By contrast, they provide no evidence to support the fluency-first hypothesis. Thus, our findings suggest that the interplay among sense-making competencies and perceptual fluency is not mutual: sense-making competencies enhance the acquisition of fluency, but fluency may prevent students from engaging in productive sense-making processes that lead to the acquisition of sense-making competencies. Further research pending, we may conclude that, to enhance learning of domain knowledge, instruction should provide students with sense-making activities before fluency-building activities. This recommendation is in line with what educational practice guides implicitly advocate by requiring sense-making competencies earlier than fluency with representations (NCTM 2006; Siegler et al. 2010). Given that not all predictions by the sense-making-first hypothesis were confirmed, our conclusions remain somewhat tentative and should be replicated. Hence, we hope that our findings inspire more research to investigate whether, indeed, sense-making competencies prepares students to acquire perceptual fluency. Because representational competencies are critical to students' learning of domain knowledge and because many STEM domains employ multiple GRs to emphasize complementary aspects of the domain knowledge, such research will likely have broad impact in many domains.

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References

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*(3), 183–198. doi:10.1016/j.learninstruc.2006.03.001.

Ainsworth, S. (2008). How should we evaluate multimedia learning environments? In *Understanding multimedia documents* (pp. 249–265). New York: Springer.

Ainsworth, S., Bibby, P., & Wood, D. (2002). Examining the effects of different multiple representational systems in learning primary mathematics. *Journal of the Learning Sciences, 11*(1), 25–61. doi:10.1207/S15327809JLS1101_2.

Ainsworth, S., & Loizou, A. (2003). The effects of self-explaining when learning with text or diagrams. *Cognitive Science: A Multidisciplinary Journal, 27*(4), 669–681.

Airey, J., & Linder, C. (2009). A disciplinary discourse perspective on university science learning: Achieving fluency in a critical constellation of modes. *Journal of Research in Science Teaching, 46*(1), 27–49. doi:10.1002/tea.20265.

Aleven, V. (2010). Rule-based cognitive modeling for intelligent tutoring systems. In R. Nkambou, J. Bourdeau, & R. Mizoguchi (Eds.), *Studies in Computational Intelligence: Vol. 308. Advances in intelligent tutoring systems* (pp. 33–62). Berlin: Springer. doi:10.1007/978-3-642-14363-2_3.

Aleven, V. A. W. M. M., & Koedinger, K. R. (2002). An effective metacognitive strategy: Learning by doing and explaining with a computer-based Cognitive Tutor. *Cognitive Science, 26*(2), 147–179. doi:10.1016/S0364-0213(02)00061-7.

Aleven, V., McLaren, B. M., Sewall, J., & Koedinger, K. R. (2009). A new paradigm for intelligent tutoring systems: Example-tracing tutors. *International Journal of Artificial Intelligence in Education, 19*(2), 105–154.

Aleven, V., McLaren, B. M., Sewall, J., van Velsen, M., Popescu, O., Demi, S., … Koedinger, K. R. (2016). Example-Tracing tutors: Intelligent tutor development for non-programmers. *International Journal of Artificial Intelligence in Education*, *26*(1), 224–269. doi:10.1007/s40593-015-0088.

Anderson, J. R. (1983). *The architecture of cognition*. Cambridge, MA: Harvard University Press.

Anderson, J. R., Corbett, A. T., Koedinger, K. R., & Pelletier, R. (1995). Cognitive tutors: Lessons learned. *Journal of the Learning Sciences, 4*(2), 167–207. doi:10.1207/s15327809jls0402_2.

Atkinson, R. K., Renkl, A., & Merrill, M. M. (2003). Transitioning from studying examples to solving problems: Effects of self-explanation prompts and fading worked-out steps. *Journal of Educational Psychology, 95*(4), 774–783. doi:10.1037/0022-0663.95.4.774.

Baetge, I., & Seufert, T. (2010). *Effects of support for coherence formation in computer science education*. Paper presented at the EARLI SIG 6/7, Ulm.

Berthold, K., Eysink, T. H. S., & Renkl, A. (2008). Assisting self-explanation prompts are more effective than open prompts when learning with multiple representations. *Instructional Science, 27*(4), 345–363. doi:10.1007/s11251-008-9051-z.

Berthold, K., & Renkl, A. (2009). Instructional aids to support a conceptual understanding of multiple representations. *Journal of Educational Research, 101*(1), 70–87. doi:10.1037/a0013247.

Bieda, K. N., & Nathan, M. J. (2009). Representational disfluency in algebra: Evidence from student gestures and speech. *ZDM*, *41*(5), 637–650.

Bodemer, D., & Faust, U. (2006). External and mental referencing of multiple representations. *Computers in Human Behavior, 22*(1), 27–42. doi:10.1016/j.chb.2005.01.005.

Bodemer, D., Ploetzner, R., Bruchmüller, K., & Häcker, S. (2005). Supporting learning with interactive multimedia through active integration of representations. *Instructional Science, 33*(1), 73–95. doi:10.1007/s11251-004-7685-z.

Bodemer, D., Ploetzner, R., Feuerlein, I., & Spada, H. (2004). The active integration of information during learning with dynamic and interactive visualisations. *Learning and Instruction, 14*(3), 325–341. doi:10.1016/j.learninstruc.2004.06.006.

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics, 64*(3), 293–316. doi:10.1007/s10649-006-9036-2.

Chickering, D. M. (2002). Optimal structure identification with greedy search. *Journal of Machine Learning Research, 3,* 507–554. doi:10.1162/153244303321897717.

Cramer, K. (2001). Using models to build an understanding of functions. *Mathematics teaching in the middle school, 6*(5), 310–318.

de Jong, T., Ainsworth, S., Dobson, M., Van der Meij, J., Levonen, J., Reimann, P., … Swaak, J. (1998). Acquiring knowledge in science and mathematics: The use of multiple representations in technology-based learning environments. In M. W. Van Someren, W. Reimers, H. P. A. Boshuizen, & T. de Jong (Eds.), *Learning with multiple representations* (pp. 9–41). Bingley: Emerald Group.

DeLoache, J. S. (2000). Dual representation and young children's use of scale models. *Child Development, 71*(2), 329–338. doi:10.1111/1467-8624.00148.

Dreher, A., & Kuntze, S. (2014). Teachers facing the dilemma of multiple representations being aid and obstacle for learning: Evaluations of tasks and theme-specific noticing. *Journal für Mathematik-Didaktik*. doi:10.1007/s13138-014-0068-3.

Dreyfus, H., & Dreyfus, S. E. (1986). Five steps from novice to expert. In *Mind over machine: The power of human intuition and expertise in the era of the computer* (pp. 16–51). New York: The Free Press.

Eilam, B., & Ben-Peretz, M. (2012). *Teaching, learning, and visual literacy: The dual role of visual representation*. New York: Cambridge University Press.

Gadgil, S., Nokes-Malach, T. J., & Chi, M. T. (2012). Effectiveness of holistic mental model confrontation in driving conceptual change. *Learning and Instruction, 22*(1), 47–61. doi:10.1016/j.learninstruc.2011.06.002.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science, 7*(2), 155–170. doi:10.1207/s15516709cog0702_3.

Gentner, D., Loewenstein, J., & Thompson, L. (2003). Learning and transfer: A general role for analogical encoding. *Journal of Educational Psychology, 95*(2), 393–405. doi:10.1037/0022-0663.95.2.393.

Gibson, E. J. (1969). *Principles of perceptual learning and development*. New York: Prentice Hall.

Gibson, E. J. (2000). Perceptual learning in development: Some basic concepts. *Ecological Psychology, 12*(4), 295–302. doi:10.1207/S15326969ECO1204_04.

Gibson, B., Zhu, X., Rogers, T., Kalish, C., & Harrison, J. (2010). Humans learn using manifolds, reluctantly. In *Advances in neural information processing systems* (Vol. 24).

Gilbert, J. K. (2005). Visualization: A metacognitive skill in science and science education. In J. K. Gilbert (Ed.), *Visualization: Theory and practice in science education* (pp. 9–27). Dordrecht: Springer.

Gilbert, J. K. (2008). Visualization: An emergent field of practice and inquiry in science education. In J. K. Gilbert, M. Reiner, & M. B. Nakhleh (Eds.), *Visualization: Theory and practice in science education* (Vol. 3, pp. 3–24). Dordrecht: Springer.

Glenberg, A. M., Wilkinson, A. C., & Epstein, W. (1982). The illusion of knowing: Failure in the self-assessment of comprehension. *Memory & Cognition, 10*(6), 597–602. doi:10.3758/BF03202442.

Goldstone, R. L., & Barsalou, L. W. (1998). Reuniting perception and conception. *Cognition, 65*(2), 231–262. doi:10.1016/S0010-0277(97)00047-4.

Gutwill, J. P., Frederiksen, J. R., & White, B. Y. (1999). Making their own connections: Students' understanding of multiple models in basic electricity. *Cognition and Instruction, 17*(3), 249–282. doi:10.1207/S1532690XCI1703_2.

Johnson, C. I., & Mayer, R. E. (2010). Applying the self-explanation principle to multimedia learning in a computer-based game-like environment. *Computers in Human Behavior, 26*(6), 1246–1252.

Jones, L. L., Jordan, K. D., & Stillings, N. A. (2005). Molecular visualization in chemistry education: The role of multidisciplinary collaboration. *Chemistry Education Research and Practice, 6*(3), 136–149. doi:10.1039/B5RP90005K.

Kahneman, D. (2003). A perspective on judgment and choice—Mapping bounded rationality. *American Psychologist, 9,* 697–720. doi:10.1037/0003-066X.58.9.697.

Kaput, J. (1987). Towards a theory of symbol use in mathematics. In C. Janvier (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 159–195). Mahwah, NJ: Lawrence Erlbaum Associates.

Kellman, P. J., & Massey, C. M. (2013). Perceptual learning, cognition, and expertise. In B. H. Ross (Ed.), *The psychology of learning and motivation* (Vol. 558, pp. 117–165). New York: Elsevier Academic Press.

Kellman, P. J., Massey, C. M., Roth, Z., Burke, T., Zucker, J., Saw, A., … Wise, J. (2008). Perceptual learning and the technology of expertise: Studies in fraction learning and algebra. *Pragmatics & Cognition, 16*(2), 356–405. doi:10.1075/pc.16.2.07kel.

Kellman, P. J., Massey, C. M., & Son, J. Y. (2009). Perceptual learning modules in mathematics: Enhancing students' pattern recognition, structure extraction, and fluency. *Topics in Cognitive Science, 2*(2), 285–305. doi:10.1111/j.1756-8765.2009.01053.x.

Kieren, T. E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research*. Hillsdale, NJ: Erlbaum.

Koedinger, K. R., Baker, R., Cunningham, K., Skogsholm, A., Leber, B., & Stamper, J. (2010). A data repository for the EDM community: The PSLC Data-Shop. In C. Romero (Ed.), *Handbook of educational data mining* (pp. 10–12). Boca Raton, FL: CRC Press.

Koedinger, K. R., Corbett, A. T., & Perfetti, C. (2012). The knowledge-learning-instruction framework: Bridging the science-practice chasm to enhance robust student learning. *Cognitive Science, 36*(5), 757–798. doi:10.1111/j.1551-6709.2012.01245.x.

Kozma, R., & Russell, J. (2005). Students becoming chemists: Developing representational competence. In J. Gilbert (Ed.), *Visualization in science education* (pp. 121–145). Dordrecht: Springer.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33,* 159–174.

Lindgren, R. (2012). Generating a learning stance through perspective-taking in a virtual environment. *Computers in Human Behavior, 28*(4), 1130–1139. doi:10.1016/j.chb.2012.01.021.

Lowe, R. K. (1993). Constructing a mental representation from an abstract technical diagram. *Learning and Instruction, 3*(3), 157–179. doi:10.1016/0959-4752(93)90002-H.

Lowe, R. K. (1994). Selectivity in diagrams: Reading beyond the lines. *Educational Psychology, 14*(4), 467–491. doi:10.1080/0144341940140408.

Massey, C. M., Kellman, P. J., Roth, Z., & Burke, T. (2011). Perceptual learning and adaptive learning technology—Developing new approaches to mathematics learning in the classroom. In N. L. Stein & S. W. Raudenbush (Eds.), *Developmental cognitive science goes to school* (pp. 235–249). New York: Routledge.

Matthews, P. G., & Chesney, D. L. (2015). Fractions as percepts? Exploring cross-format distance effects for fractional magnitudes. *Cognitive Psychology, 78,* 28–56. doi:10.1016/j.cogpsych.2015.01.006.

McElhaney, K. W., Chang, H. Y., Chiu, J. L., & Linn, M. C. (2015). Evidence for effective uses of dynamic visualisations in science curriculum materials. *Studies in Science Education, 51*(1), 49–85. doi:10.1080/03057267.2014.984506.

Nathan, M. J., Walkington, C., Boncoddo, R., Pier, E. L., Williams, C. C., & Alibali, M. W. (2014). Actions speak louder with words: The roles of action and pedagogical language for grounding mathematical reasoning. *Learning and Instruction, 33,* 182–193. doi:10.1016/j.learninstruc.2014.07.001.

National Council of Teachers of Mathematics, NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics, NCTM. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: National Council of Teachers of Mathematics.

National Mathematics Advisory Board Panel, NMAP. (2008). *Foundations for success: Report of the National Mathematics Advisory Board Panel*. Washington, DC: U.S. Government Printing Office.

National Research Council, NRC. (2006). *Learning to think spatially*. Washington, DC: National Academies Press.

Ohlsson, S. (2008). Computational models of skill acquisition. In R. Sun (Ed.), *The Cambridge handbook of computational psychology* (pp. 359–395). New York: Cambridge University Press.

Ozcelik, E., Karakus, T., Kursun, E., & Cagiltay, K. (2009). An eye-tracking study of how color coding afffects multimedia learning. *Computers & Education, 53*(2), 445–453. doi:10.1016/j.compedu.2009.03.002.

Özgün-Koca, S. A. (2008). Ninth grade students studying the movement of fish to learn about linear relationships: The use of video-based analysis software in mathematics classrooms. *The Mathematics Educator, 18*(1), 15–25.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation (s) in developing mathematical understanding. *Theory into Practice, 40*(2), 118–127. doi:10.1207/s15430421tip4002_6.

Patel, Y., & Dexter, S. (2014). Using multiple representations to build conceptual understanding in science and mathematics. In M. Searson & M. Ochoa (Eds.), *Proceedings of society for information technology & teacher education international conference 2014* (Vol. 2014, pp. 1304–1309). Chesapeake, VA: AACE.

Post, T. R., Behr, M. J., & Lesh, R. (1982). Interpretations of rational number concepts. In L. Silvey & J. R. Smart (Eds.), *Mathematics for the middle grades* (pp. 5–9). Reston, VA: National Council of Teachers of Mathematics.

Rau, M. A., Aleven, V., Rummel, N., & Rohrbach, S. (2013). Why interactive learning environments can have it all: Resolving design conflicts between conflicting goals. In *Proceedings of the SIGCHI 2013 ACM conference on human factors in computing systems* (pp. 109–118). New York: ACM.

Rau, M. A., Aleven, V., Rummel, N., & Pardos, Z. (2014a). How should intelligent tutoring systems sequence multiple graphical representations of fractions? A multi-methods study. *International Journal of Artificial Intelligence in Education*, *24*(2), 125–161. doi:10.1007/s40593-013-0011-7.

Rau, M. A., Aleven, V., & Rummel, N. (2014b). Sequencing sense-making and fluency-building support for connection making between multiple graphical representations. In J. L. Polman, E. A. Kyza, D. K. O'Neill, I. Tabak, W. R. Penuel, A. S. Jurow, et al. (Eds.), *Learning and becoming in practice: The international conference of the learning sciences (ICLS) 2014* (Vol. 2, pp. 977–981). Boulder, CO: International Society of the Learning Sciences.

Rau, M. A., Aleven, V., & Rummel, N. (2015). Successful learning with multiple graphical representations and self-explanation prompts. *Journal of Educational Psychology*, *107*(1), 30–46. doi:10.1037/a0037211.

Rau, M. A. (2016). Conditions for the effectiveness of multiple visual representations in enhancing STEM learning. *Educational Psychology Review*. doi:10.1007/s10648-016-9365-3.

Rau, M. A., Aleven, V., & Rummel, N. (2016). Supporting students in making sense of connections and in becoming perceptually fluent in making connections among multiple graphical representations. *Journal of Educational Psychology*. doi:10.1037/edu0000145.

Renkl, A. (2005). The worked-out example principle in multimedia learning. In R. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 229–246). Cambridge: Cambridge University Press.

Richman, H. B., Gobet, F., Staszewski, J. J., & Simon, H. A. (1996). Perceptual and memory processes in the acquisition of expert performance: The EPAM model. In K. A. Ericsson (Ed.), *The road to excellence? The acquisition of expert performance in the arts and sciences, sports and games* (pp. 167–187). Mahwah, NJ: Erlbaum Associatees.

Schnotz, W. (2005). An integrated model of text and picture comprehension. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 49–69). New York: Cambridge University Press.

Schnotz, W. (2014). An integrated model of text and picture comprehension. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (2nd ed., pp. 72–103). New York: Cambridge University Press.

Seufert, T. (2003). Supporting coherence formation in learning from multiple representations. *Learning and Instruction, 13*(2), 227–237. doi:10.1016/S0959-4752(02)00022-1.

Seufert, T., & Brünken, R. (2006). Cognitive load and the format of instructional aids for coherence formation. *Applied Cognitive Psychology, 20,* 321–331. doi:10.1002/acp.1248.

Siegler, R. S., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., … Wray, J. (2010). *Developing effective fractions instruction: A practice guide*. Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Spirtes, P., Glymour, C., & Scheines, R. (2000). *Causation, prediction, and search* (2nd ed.). Cambridge, MA: MIT.

Stern, E., Aprea, C., & Ebner, H. G. (2003). Improving cross-content transfer in text processing by means of active graphical representation. *Learning and Instruction, 13*(2), 191–203. doi:10.1016/S0959-4752(02)00020-8.

Taber, K. S. (2013). Revisiting the chemistry triplet: Drawing upon the nature of chemical knowledge and the psychology of learning to inform chemistry education. *Chemistry Education Research and Practice, 14*(2), 156–168. doi:10.1039/C3RP00012E.

Uttal, D. H., & O'Doherty, K. (2008). Comprehending and learning from 'visualizations': A developmental perspective. In J. Gilbert (Ed.), *Visualization: Theory and practice in science education* (pp. 53–72). Dordrecht: Springer.

van der Meij, J., & de Jong, T. (2006). Supporting students' learning with multiple representations in a dynamic simulation-based learning environment. *Learning and Instruction, 16*(3), 199–212. doi:10.1016/j.learninstruc.2006.03.007.

van der Meij, J., & de Jong, T. (2011). The effects of directive self-explanation prompts to support active processing of multiple representations in a simulation-based learning environment. *Journal of Computer Assisted learning, 27*(5), 411–423. doi:10.1111/j.1365-2729.2011.00411.x.

Van Gog, T., Paas, F., Van Merriënboer, J. J. G., & Witte, P. (2005). Uncovering the problem-solving process: Cued retrospective reporting versus concurrent and retrospective reporting. *Journal of Experimental Psychology, 11,* 237–244. doi:10.1037/1076-898X.11.4.237.

VanLehn, K. (2011). The relative effectiveness of human tutoring, intelligent tutoring systems and other tutoring systems. *Educational Psychologist, 46*(4), 197–221. doi:10.1080/00461520.2011.611369.

Wertsch, J. V., & Kazak, S. (2011). Saying more than you know in instructional settings. In T. Koschmann (Ed.), *Theories of learning and studies of instructional practice* (pp. 153–166). New York: Springer.

Wise, J. A., Kubose, T., Chang, N., Russell, A., & Kellman, P. J. (2000). Perceptual learning modules in mathematics and science instruction. In P. Hoffman & D. Lemke (Eds.), *Teaching and learning in a network world* (pp. 169–176). Amsterdam: IOS Press.

Wu, H. K., & Shah, P. (2004). Exploring visuospatial thinking in chemistry learning. *Science Education, 88*(3), 465–492. doi:10.1002/sce.10126.